PERFORMANCE OF INTERLEAVED ERROR CORRECTION CODES WITH BURST ERRORS

Roy D. CIDECIYAN, Simeon FURRER and Mark A. LANTZ
IBM Research - Zurich, CH-8803 Rueschlikon, Switzerland

I. INTRODUCTION

Markov chains are natural models for many communication and storage channels with memory in which the channel condition, characterized by a state variable, changes with time. Two-state Markov chains with a good state and a bad state, proposed by Gilbert and Elliott, have been widely used to model channels with burst errors. The performance of interleaved error correction codes (ECC) on channels with memory have been analyzed for a fixed symbol size, e.g., assuming a simplified Gilbert channel for symbol errors [1, 2].

Here, we present a Gilbert-Elliott channel for symbol errors to analyze ECC performance of interleaved block codes with burst errors. In contrast to previous approaches, the proposed Gilbert-Elliott channel for symbol errors is based on a simplified Gilbert channel for bit errors, which offers several advantages: 1) it is readily characterized by measuring average raw bit-error rate (BER) and average bit-error burst length at the output of the detector, and 2) it enables direct comparisons of the performance of block codes with different symbol sizes. An exact expression and a tight lower bound for the codeword error probability are derived for the proposed Gilbert-Elliott channel for symbol errors. Error floors, which are observed when the average raw BER is low, are analyzed.

II. PERFORMANCE ANALYSIS AND ERROR FLOORS

A simplified Gilbert channel for bit errors is characterized by a 2×2 state transition probability matrix $\boldsymbol{C}$, where $\boldsymbol{C} = \begin{bmatrix} b & 1-b \\ 1-a & a \end{bmatrix}$, b is the probability of staying in the good state G and a is the probability of staying in the bad state B. The bit error probability at state G is 0 whereas the bit error probability at state B is 1. The average raw BER is then $p_b = (1-b)/(2-a-b)$. The row vector $\boldsymbol{\pi} = [\pi_G, \pi_B]$ is defined by $\boldsymbol{\pi}\boldsymbol{C} = \boldsymbol{\pi}$, where $\pi_G = 1- p_b$ is the stationary probability of being at state G and $\pi_B = p_b$ is the stationary probability of being at state B. In a simplified Gilbert model, the distribution of occupancy times for both states G and B is geometric with means $(1-b)^{-1}$ and $(1-a)^{-1}$, respectively. In the following, a is a fixed constant for a given channel with burst errors and b changes as a function of a and p_b. Therefore, the simplified Gilbert channel can be characterized either by a and b or by a and p_b, i.e., measurement of the average raw BER p_b and the average length of bit-error bursts $(1-a)^{-1}$ at the output of the detector is sufficient for channel identification. In our channel model with burst errors, the average bit-error burst length $(1-a)^{-1}$ is fixed whereas the average length of error-free intervals $(1-b)^{-1} = (1-a)^{-1}(1- p_b)/p_b$ increases as the average raw BER decreases with increasing signal-to-noise ratio (SNR).

The Markov chain characterizing s-bit symbol errors, can be specified by the 2×2 s-step state transition probability matrix $\boldsymbol{T} \overset{\text{def}}{=} \boldsymbol{C}^s \overset{\text{def}}{=} \begin{bmatrix} B & 1-B \\ 1-A & A \end{bmatrix}$. The symbol error probability at state G is $(1-B_1)$ with $B_1 = b^s$, whereas the symbol error probability at state B is $(1-A_1)$ with $A_1 = (1-a)b^{s-1}$. Clearly, the two-state Markov chain describing symbol errors is a Gilbert-Elliott channel and the average symbol error probability is $p_s = (1- p_b)(1-B_1) + p_b(1-A_1)$. In the following, we assume that the error correction code has N s-bit symbols and an error correction capability of t symbols. Furthermore, it is assumed that the symbol interleaving depth is I and the interleaved codewords are transmitted over the previously described Gilbert-Elliott channel. The codeword error probability can be computed by $p_C = \sum_{m=t+1}^{N} P(m,N)$, where $P(m,N)$ is the probability of m erroneous symbols in an N-symbol codeword. It can be shown that $P(m,N) = \langle \boldsymbol{\pi}(\boldsymbol{T}^{I-1}\boldsymbol{E}(x))^N \boldsymbol{1}\rangle_m$, where the m-th coefficient c_m of a polynomial $f(x) = \sum_{m=0}^{n} c_m x^m$ is denoted by $c_m \overset{\text{def}}{=} \langle f(x)\rangle_m$, $\boldsymbol{1}$ is a 2×1 column vector of ones, x is an indeterminate, and $\boldsymbol{E}(x)$ is a 2×2 matrix that labels each state transition for counting purposes by the probability of making no symbol error plus the probability of making a symbol error multiplied by the counting variable x. It can be shown that $\boldsymbol{E}(x) = \begin{bmatrix} B_1 + (B - B_1)x & (1-B)x \\ A_1 + (1-A-A_1)x & Ax \end{bmatrix}$.

R. D. CIDECIYAN
IBM Research – Zurich
Saeumerstr. 4, Rueschlikon, Switzerland
tel: +41 44 724 8523
e-mail: cid@zurich.ibm.com

A tight lower bound p_L on the codeword error probability has been derived for interleaved codewords. Specifically, $p_C > p_L = p_b a^{Its+1} D(a,b,N,t,I,s)$, where

$$D(a,b,N,t,I,s) = \frac{1-a}{a}\left(\sigma(b^{Is}, N-t)\,\sigma\left(\frac{b}{a}, s\right) + b^{Is}\sigma(b^{Is}, N-t-1)\left(\sigma\left(\frac{b}{a}, Is-s+1\right) - 1\right)\right) + 1, \quad (1)$$

and $\sigma(q,n) = (1-q^n)/(1-q)$. In the error floor regime of high SNR, $p_L \to p_F$ as $p_b \to 0$ and $b \to 1$. In this regime, there is a linear relationship between p_b and the error floor probability p_F given by

$$p_F = p_b a^{Its+1} \lim_{p_b \to 0} D(a,b,N,t,I,s) = p_b a^{Its+1}\left((N-t)(a^{-s}-1) + (N-t-1)(1-a^{Is-s}) + 1\right). \quad (2)$$

In the low SNR regime of high raw BER p_b, we approximate p_c assuming independent symbol errors with probability $p_s = (1-p_b)(1-B_1) + p_b(1-A_1)$. The autocorrelation $R(k)$ between two symbols spaced k symbols apart in a codeword is computed using Eq. (4.54) in [3]

$$R(k) = p_s^{-1}(1-p_s)^{-1}(1-A_1-p_s)(p_s-1+B_1)(A+B-1)^{Ik}. \quad (3)$$

For a typical error correction code with N=240, s=8, t=6 that is used in magnetic tape storage, and interleaving depths I=1 and I=4, Fig. 1 depicts the codeword error probability p_C as a function of raw BER p_b and a=15/16. The error floor for I=1 is about four orders higher than for I=4. For a=15/16, N=240, s=8, I=4 and t=3 and t=6, Fig. 2 shows the codeword error probability p_C as a function of raw BER p_b. The error floor for t=3 is about three orders higher than for t=6. In both figures, the high-BER approximations and low-BER error floors are close to the exact computations. For a=15/16, s=8 and symbol interleaving depths I=1 and I=4, Fig. 3 illustrates the autocorrelation of adjacent symbols in a codeword $R(1)$ vs. raw BER. $R(1)$ for I=4 is less than 0.1, which is about four times less than for I=1. Figure 4 shows that the error distribution for N=240, t=6, I=4, s=8, a=15/16 and $p_b=10^{-2}$ can be approximated by the binomial distribution for N=240 and the corresponding symbol error probability $p_s=1.4\times10^{-2}$, thus justifying the high-BER approximation.

REFERENCES

1) J. R. Yee and E. J. Weldon, *IEEE Trans. Commun.*, vol. 43, pp. 2316-2323, 1995.
2) J. Wolf, *IEEE Trans. Magn.*, vol. 34, pp. 75-79, 1998.
3) J. Huber, Codierung für gedächtnisbehaftete Kanäle, Dissertation, Bundeswehr University Munich, 1982.

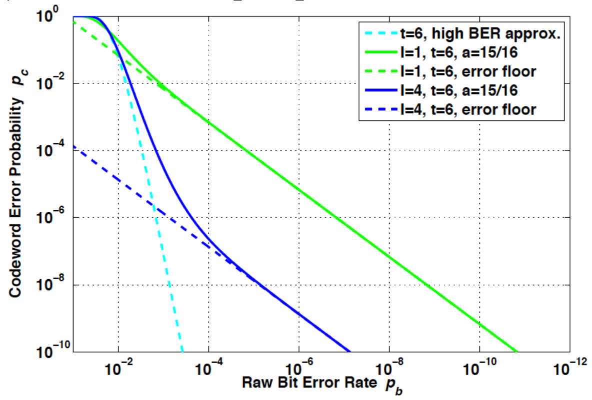

Fig. 1. Codeword error probability vs. raw BER for N=240, s=8, t=6 and interleaving depths I=1 and I=4.

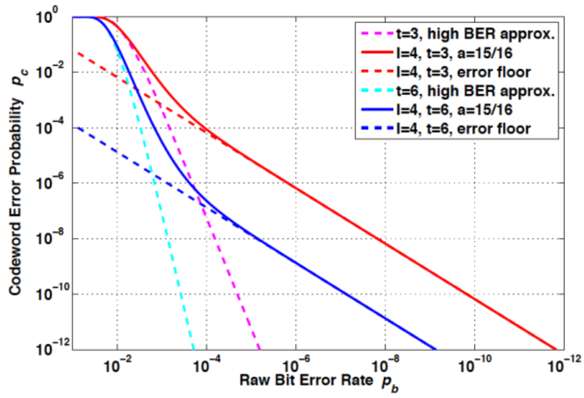

Fig. 2. Codeword error probability vs. raw BER for N=240, s=8, I=4 and ECC capabilities t=3 and t=6.

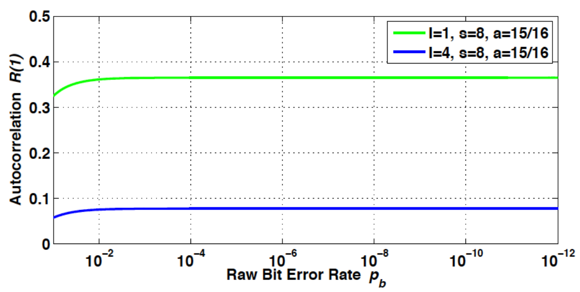

Fig. 3. Autocorrelation of adjacent symbols in a codeword vs. raw BER for interleaving I=1 and I=4.

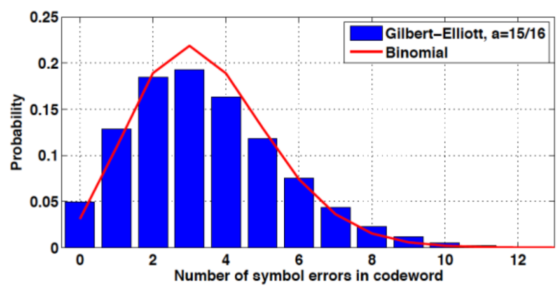

Fig. 4. Error distribution for N=240, t=6, I=4, s=8, a=15/16, $p_b=10^{-2}$ and binomial distribution.